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The Invariant Charges of the Nambu-Goto Theory in WKB-Approximation: Renormalization*

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Abstract. We discuss the one loop renormalization of the reparametrization invariant non-local conserved charges of the Nambu-Goto string theory. In addition we show the stability of a special well-known state under the corresponding infinitesimal symmetry transformations – at least in the WKB-approximation.

I. Classical Considerations

The classical Nambu-Goto string theory possesses infinitely many independent, reparametrization invariant non-local conserved charges [1] which act as infinitesimal generators of symmetry transformations. Here we want to argue in the Euclidean version that these charges can be carried over to the quantum theory as well-defined operators – at least in WKB-approximation – without introducing unfamiliar counterterms. Further, we want to demonstrate that the renormalized loop wave-functional $\psi(\mathscr{C})$ constructed by the authors of [2] is invariant under the above symmetry transformations – at least in WKB-approximation. Thus, $\psi(\mathscr{C})$ is likely to correspond to the "Euclidean" ground state of the system.

Associated with bosonic Euclidean closed strings are closed curves $\mathscr{C}$ in $\mathbb{R}^d$, $d=3,4,\ldots$. Let $x_\mu = x_\mu(\sigma) = x_\mu(\sigma+2\pi)$, $\sigma \in \mathbb{R}$ be a parametrization of $\mathscr{C}$ and let M be a constant mass parameter. Classically the invariant charges in question are given in terms of cyclic sums of path-ordered multiple integrals:

$$\begin{aligned}\mathscr{Z}^{\pm}_{\mu_1\ldots\mu_N} = &\int_0^{2\pi} d\sigma_1 \ldots \int_0^{2\pi} d\sigma_N \theta(\sigma_1-\sigma_2)\ldots\theta(\sigma_{N-1}-\sigma_N)\\ &\times\left[\frac{1}{i}\frac{\delta}{\delta x_{\mu_1}(\sigma_1)} \pm M^2 x'_{\mu_1}(\sigma_1)\right]\ldots\left[\frac{1}{i}\frac{\delta}{\delta x_{\mu_N}(\sigma_N)} \pm M^2 x'_{\mu_N}(\sigma_N)\right]\\ &+\text{ cyclic permutations of the indices } \mu_1,\ldots,\mu_N .\end{aligned}$$

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