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Algebraic Properties of the Invariant Charges of the Nambu-Goto Theory*

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Abstract. We analyse the infinite dimensional algebra of observable non-local integrals of motion of the Nambu-Goto string theory.

I. Introduction

Some time ago one of the present authors suggested a reparametrization invariant approach towards the quantization of the free relativistic closed bosonic string [1, 2]. This approach was modelled after the quantization of the free relativistic particle in terms of irreducible representations of the Poincaré algebra. In the Nambu-Goto theory [3] of the string moving in d-dimensional space-time $\mathbb{M}^d$, the analogue $\mathfrak{g}$ of the Poincaré algebra has been shown [1] to be of the following type

$$\mathfrak{g}=\mathrm{so}(1,d-1)\dot{+}(\mathbb{M}^d\oplus(\mathfrak{h}_{\mathscr{P}}^{+}\oplus\mathfrak{h}_{\mathscr{P}}^{-})),$$

where $\mathrm{so}(1,d-1)$ stands for the Lie algebra of the homogeneous Lorentz transformations, $\mathbb{M}^d$ for the Lie algebra of translations, $\mathfrak{h}_{\mathscr{P}}^{+}$ and $\mathfrak{h}_{\mathscr{P}}^{-}$ for the infinite-dimensional algebra of infinitesimal generators of certain "internal" symmetry transformations of the string. Explicitly, a basis of $\mathrm{so}(1,d-1)$ is furnished by the infinitesimal generators $M_{\mu\nu}$ of Lorentz transformations in the μ,ν plane, $\mu\neq\nu$, $\mu,\nu=0,1,\ldots,d-1$, $M_{\mu\nu}=-M_{\nu\mu}$, a basis of $\mathbb{M}^d$ by the components $\mathscr{P}_\mu,\mu=0,1,\ldots,d-1$ of the energy-momentum operator, i.e. the infinitesimal generators of translations in the μ direction, and finally a basis of $\mathfrak{h}_{\mathscr{P}}^{+}$ and $\mathfrak{h}_{\mathscr{P}}^{-}$ is furnished by certain reparametrization invariant conserved "internal", "non-local" charges $\mathscr{Z}^{\mathrm{red}+}$ and $\mathscr{Z}^{\mathrm{red}-}$ respectively. The charges $\mathscr{Z}^{\mathrm{red}+}$ and $\mathscr{Z}^{\mathrm{red}-}$ commute with the momenta $\mathscr{P}_\mu$ and transform covariantly according to finite dimensional (irreducible) representations of the Lorentz group. The elements of $\mathfrak{h}_{\mathscr{P}}^{+}$ commute with all the elements of $\mathfrak{h}_{\mathscr{P}}^{-}$.

The central idea of the new approach consists of viewing the loop equations of the Nambu-Goto theory as an infinite collection of representation conditions for

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