Commun. Math. Phys. 105, 527–545 (1986)

Communications in Mathematical Physics

Phases Coexistence and Surface Tensions for the Potts Model

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Abstract. The q states Potts model exhibits a first order phase transition at some inverse temperature β_t between "ordered" and "disordered" phases for q large as proved in [1]. In space dimension 2 we use the *duality transformation* as an *internal symmetry* of the partition function at β_t to derive an estimate on the probability of a contour. This enables us to prove the preceding result and the following new results:

(i) The discontinuity of the mass gap at β_t.

(ii) The existence of a *strictly positive* surface tension between two ordered phases up to β_t.

(iii) The existence of a non-zero surface tension between an "ordered" and the "disordered" phase at β_t.

I. Introduction

An usual situation in statistical mechanics is the coexistence of several ordered phases (corresponding to the different ground states of the Hamiltonian). Such a situation is very well described by the Pirogov-Sinai theory [8]. At high temperature there is a unique phase (the "disordered one"). We sketch two typical behaviours.

1. There is a *second order phase transition at* β_c (critical inverse temperature) and above β_c a surface tension between different ordered phases which goes to zero at β_c. (This situation occurs in many models such as the Ising model [3, 4].)

Moreover at high temperature the two points function decays exponentially in the distance with a mass gap vanishing at β_c, as shown by Simon [2] (see also previous works of Dobrushin and Pechersky [28]).

2. The phase transition is first order at β_t. Such a situation was exhibited by Dobrushin and Shlosman [22] for continuous spin and by Kotecky and Shlosman

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