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Hausdorff Dimension of Attractors for the Two-Dimensional Navier-Stokes Equations with Boundary Conditions

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Abstract. We consider a viscous incompressible fluid enclosed in a region of $\mathbb{R}^2$, and subject to boundary conditions. We obtain explicit bounds (depending only on the data) for the entropy (Kolmogorov-Sinaï invariant) and the Hausdorff dimension of attracting sets.

1. Introduction

We consider a viscous incompressible fluid enclosed in a region Ω of $\mathbb{R}^2$. The time evolution of the fluid is described by the Navier-Stokes equations with boundary conditions. Two kinds of boundary conditions are investigated: a given velocity and an imposed force on the boundary Γ of Ω.

Ruelle ([7]) has obtained rigourous bounds on the entropy (Kolmogorov-Sinaï invariant) and the Hausdorff dimension of attracting sets, involving the rate of energy dissipation in the fluid. These bounds have been improved by Lieb ([5]) and a complete statement of available results is given in [8] (see also [1, 2]). Using these estimates, we derive explicit bounds on these quantities (i.e., bounds depending only on the data).

2. Given Velocity on the Boundary

Let Ω a bounded open region of $\mathbb{R}^2$, with a C^3 boundary Γ. Let $\varphi \in H^{3/2}(\Gamma)^2$ [we recall $H^{3/2}(\Gamma)=\gamma_0 H^2(\Omega)$, where the linear operator γ_0 is defined on $H^1(\Omega)$ by $\gamma_0 u = u|_\Gamma$] such that $\int_\Gamma \varphi \cdot n \, d\Gamma = 0$, n being the unit outward normal on Γ. The evolution of a viscous fluid enclosed in Ω, subject to the boundary condition φ, is

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