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On the Boltzmann Equation in Unbounded Space far from Equilibrium, and the Limit of Zero Mean Free Path

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Abstract. This paper studies Loeb solutions of the Boltzmann equation in unbounded space under natural initial conditions of finite mass, energy, and entropy. An existence theory for large initial data is presented. Maxwellian behaviour is obtained in the limits of zero mean free path and of infinite time. In the standard, space-homogeneous, hard potential case, the infinite time limit is of strong L^1 type.

1. Preliminaries

This paper considers Boltzmann's equation, a prototype model of rarefied neutral gases driven by binary collisions. For gases in full space or bounded containers, various contraction mapping estimates can be used to prove the existence of unique, smooth solutions converging to an equilibrium with time, if the initial values are close enough to this equilibrium. Such methods break down, when the initial values are further away from equilibrium, and so do natural compactness arguments, at least in the space-dependent case. For another approach, recall that what happens at distances or within volumes below, say, the scale of elementary particle phenomena, is an artefact of the model with little direct experimental relevance. In this perspective, the question whether the model starts from an underlying set of rationally, really, or infinitesimally spaced points, should be decided purely on mathematical grounds.

In this paper we study the Boltzmann gases on a three-dimensional continuum filled with a denser set of points than the usual triples of reals, in that a non-standard extension of the reals is used instead. In such a setting an integrated form of the actual Boltzmann equation can be solved for arbitrary initial mass distributions with finite initial entropy and second moments. Just like all classical computational models, whatever interesting physical quantities there are, such as moments, they correspond also in the present context to real-valued integrals of the solutions multiplied by test functions. Our solutions display a weak form of Maxwellian behaviour in the small mean free path limit. Globally bounded moments of some