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Global Validity of the Boltzmann Equation for a Two-Dimensional Rare Gas in Vacuum

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Abstract. We consider a system of N hard disks in $\mathbb{R}^2$ in the Boltzmann–Grad limit (i.e. $N \to \infty$, $d \searrow 0$, $N \cdot d \to \lambda^{-1} > 0$, where d is the diameter of the disks). If λ is sufficiently small and if the joint distribution densities factorize at time zero, we prove that the time-evolved one-particle distribution converges for all times to the solution of the Boltzmann equation with the same initial datum.

1. Introduction

It is generally believed that in certain limit situations the dynamics of a gas of particles can be described by the Boltzmann equation. One of the basic problems in the foundations of kinetic theory is to prove the validity of this statement in a rigorous way, assuming, as a starting point, the laws of classical mechanics. The difficulty and appeal of this problem stem from the necessity that one has to relate two evolutions with very different natures: Newtonian dynamics, which are deterministic and reversible, and Boltzmann dynamics, which have a stochastic character and are irreversible.

A first result in this direction was obtained by Lanford [1] who deduced, in a rigorous way, the validity of the Boltzmann equation for short times (on the order of magnitude of a fraction of the mean free time).

In this paper we consider a two-dimensional system of hard disks and prove, by following the general strategy proposed in [1], the validity of the Boltzmann equation for all times in the case of a gas allowed to expand into free space and for large enough mean free paths (in comparison with the initial datum). In doing this we use the idea developed in [2] for solving the Cauchy problem for the Boltzmann equation in the corresponding three-dimensional situation-namely, that free flow has good contracting properties on functions rapidly decreasing at infinity.

We discuss the limitations of our result. The method applies to cases in which the density decays at infinity, so thermodynamical situations elude our analysis. Furthermore, we are not able to treat more realistic three-dimensional systems. This is probably a technical difficulty. Finally, we assume that the gas be rarefied in the