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# Instantons and Jumping Lines

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**Abstract.** We study the behaviour under deformation of holomorphic bundles of rank 2 over $\mathbb{P}_1(\mathbb{C})$. This is then applied to the description of the moduli space $\tilde{M}_n$ of framed SU(2) instantons of charge $n$; $\tilde{M}_n$ is shown to map to $\mathbb{C}^n$, with equidimensional fibers. We use this to provide a stratification of $\tilde{M}_n$ and compute the strata explicitly to codimension 4. This then yields $\pi_1(\tilde{M}_n)=\mathbb{Z}_2$, and, for the standard moduli space $M_n$, $\pi_1(M_n)=0$ for $n$ odd, $\mathbb{Z}_2$ for $n$ even.

## 1. Introduction

By twistor methods, instantons are known to be equivalent to holomorphic vector bundles on $\mathbb{P}_3(\mathbb{C})$ [1]; using the monad construction of Horrocks [12], a description of all solutions was given in [3]. Still, very little is known about the moduli space of solutions; recent work of Donaldson [6] has, however, reduced the problem to classifying certain semi-stable bundles of zero first Chern class on $\mathbb{P}_2=\mathbb{P}_2(\mathbb{C})$.

It is then natural to try to use this to classify instantons. It turns out that a convenient method for doing this is to restrict the bundle again, to the family of $\mathbb{P}_1(\mathbb{C})$'s in $\mathbb{P}_2$ through a fixed point, and to study the behaviour of the bundle as one varies the $\mathbb{P}_1$ in the family. The purpose of this article is thus twofold: to examine the behaviour under deformation of holomorphic vector bundles over $\mathbb{P}_1=\mathbb{P}_1(\mathbb{C})$, and to apply the information gained to the classification of semi-stable vector bundles over $\mathbb{P}_2$ ("*bundle*" is to be taken throughout to mean "*holomorphic bundle*"; all the results here concern the classification of *holomorphic structures*).

We have restricted our attention to bundles of rank two, which correspond to the gauge group SU(2). We obtain a description of the moduli of SU(2) instantons; it complements the monad theoretic work of Barth [5] on stable 2-bundles, but is more geometric in nature; it has the advantage of being concrete enough for us to compute, for example, the fundamental group.

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