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An Example of Absence of Turbulence for any Reynolds Number

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Abstract. We consider a viscous incompressible fluid moving in a two-dimensional flat torus. We show a particular external force $\underline{f}_0$ for which there is a globally attractive stationary state for any Reynolds number R. Moreover, for any fixed R, this stability property holds also for a neighbourhood of $\underline{f}_0$.

We consider a viscous incompressible fluid moving in a two-dimensional flat torus. The Navier-Stokes equations governing the motion are

$$\frac{\partial \underline{u}}{\partial t}+(\underline{u}\cdot\underline{\nabla})\underline{u}=-\underline{\nabla}p+\underline{f}+\nu\Delta\underline{u}, \qquad \underline{u}(0)=\underline{u}_0, \tag{1}$$

$$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}=0, \tag{2}$$

$$\int_{T^2}\underline{u}d\underline{x}=0, \qquad \int_{T^2}\underline{f}d\underline{x}=0, \tag{3}$$

$$T^2=[0,2\pi]\times[0,2\pi], \qquad \underline{x}\equiv(x,y)=x\underline{e}_1+y\underline{e}_2\in T^2,$$

where $\underline{u}(\underline{x},t)$ is the velocity, $p(\underline{x},t)\in\mathbb{R}$ the pressure, $\nu>0$ the viscosity, $\underline{f}(\underline{x})$ the external force. All functions involved are periodic in x, y of period 2π.

In our problem we fix a time scale and we assume as a reasonable Reynolds number

$$R=\sup_{\underline{x}\in T^2}|\underline{f}(\underline{x})|/\nu.$$

In general the behavior of the solutions depends on R: if R is small there exists a stationary state stable and attractive. When R increases this state loses its stability and, for large R, the motion becomes chaotic. This fact is related with the turbulence. (On this subject there is a lot of literature: see for instance [1].)

In this paper we want to show particular forces $\underline{f}_0(\underline{x})$ for which the stationary state remains attractive for every Reynolds number R. These forces are not

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