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On the Connection Between Quantum Fields and von Neumann Algebras of Local Operators

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Abstract. The relationship between a standard local quantum field and a net of local von Neumann algebras is discussed. Two natural possibilities for such an association are identified, and conditions for these to obtain are found. It is shown that the local net can naturally be so chosen that it satisfies the Special Condition of Duality. The notion of an intrinsically local field operator is introduced, and it is shown that such an operator defines a local net with which the field is locally associated. A regularity condition on the field is formulated, and it is shown that if this condition holds, then there exists a unique local net with which the field is locally associated if and only if the field algebra contains at least one intrinsically local operator. Conditions under which a field and other fields in its Borchers class are associated with the same local net are found, in terms of the regularity condition mentioned.

1. Introduction

In the attempts to formulate a mathematically satisfactory theory of particles consistent with special relativity and incorporating the notion of locality, two main approaches stand out. One of these is the general theory of (finite-component) local quantum fields [21, 28] and the other is the algebraic relativistic quantum theory [16, 1, 17, 7]. In the latter theory the primary object of interest is a net of algebras of local observables, and experience has shown that such a theory provides a suitable framework for the analysis of the general structure of a relativistically covariant, local quantum theory. Quantum field theory deals with operator-valued distributions and algebras of closable, but in general unbounded operators. The study of such objects entails considerable technical difficulties involving domain of definition questions. In spite of this, the notion of a local

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