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A Classification of Open String Models

W. Nahm

Physikalisches Institut der Universität Bonn, Nussallee 12, D-5300 Bonn 1, Federal Republic of Germany

Abstract. Open string models are classified using modular invariance. No good candidates for new models are found, though the existence of an E_8 invariant model in $R^{17,1}$, a similar one in $R^{5,1}$ and of a supersymmetric model in $R^{2,1}$ cannot be excluded by this technique. An intriguing relation between the left moving and right moving sectors of the heterotic string emerges.

Due to the conformal invariance of string theories, quantities defined by integrals over function spaces defined on the string world sheet can be described by holomorphic forms in Teichmüller space, i.e. the space of classes of conformally equivalent metrics on a compact Riemann surface of given genus. For genus $g=1,2,3$ this space is given by the coset space $H(g)/\mathrm{Sp}(g,\mathbb{Z})$, where $H(g)$ is the Siegel upper half plane of dimension $g(g+1)/2$. Thus one obtains holomorphic forms on $H(g)$ which are invariant under $\mathrm{Sp}(g,\mathbb{Z})$.

In particular this applies to various partition functions using light cone variables which can be defined as suitable traces or supertraces in the Hilbert space of a non-interacting string [1, 2]. Such a trace is given by a functional integral with the condition that the states of the string at light cone times t_0 and t_1 are the same. For closed strings a state at time t is specified by functions $f(\sigma, t)$, $\sigma \in R \bmod 2\pi$, but for constant σ_0 the functions $f(\sigma, t)$ and $f(\sigma+\sigma_0, t)$ denote the same state. This introduces a complication which will not be discussed in the present paper, which deals exclusively with open strings.

Open strings are described by functions $f(\sigma, t)$, $\sigma \in [0, \pi]$. For functional integrals yielding traces one must have $f(\sigma, t_0)=f(\sigma, t_1)$, as the boundaries at $\sigma=0, \pi$ are fixed. The string surface over which one integrates then is the annulus $[0, \pi] \times (R \bmod (t_1-t_0))$. However, one can go over to a compact double cover of this annulus, namely the torus $(R \bmod 2\pi) \times (R \bmod (t_1-t_0))$, with projection $\sigma \rightarrow |\sigma|$. Physically, the double covering corresponds to the separation of right moving and left moving excitations of the string, which due to the conformal invariance do not interact at all, apart from the boundaries, where they are reflected and transform into each other.