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Comments

Absence of Crystalline Ordering in Two Dimensions

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Abstract. We give conditions on the potential of a classical particle system, which imply absence of crystalline ordering in two dimensions. We thereby correct and extent some results in a previous paper.

1. Introduction

In an earlier paper [1] we gave a modified version of Mermin's argument for the absence of crystalline ordering in two-dimensional classical systems of point particles, [2]. In this note we would like to clarify and correct our discussion in [1], following Theorem 1 in that paper, and describe the kind of potentials for which our results apply. Since this note is a complement to [1], we use the same notations as in [1], and we do not repeat the basic definitions.

2. Relative Entropy Argument

We consider a system of point particles in $\mathbb{R}^2$. The configurations of the system are identified with the subsets, ω, of $\mathbb{R}^2$ which are locally finite: $x \in \omega$ means that there is a particle at x, and, for any bounded set Λ, $\omega_\Lambda = \omega \cap \Lambda$ is a finite subset of $\mathbb{R}^2$. The interaction is given by a two-body translation invariant potential $\phi: \mathbb{R}^2 \to \mathbb{R}$, $\phi(x) = \phi(-x)$, and we suppose that

A) ϕ is stable and regular;

B) ϕ is of class C^2, except at the origin.

The energy of a particle at x in the configuration ω is

$$H_\phi(x|\omega) \equiv H(x|\omega) = \sum_{\substack{y: x \neq y \\ y \in \omega}} \phi(x-y), \tag{2.1}$$

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