Commun. Math. Phys. 104, 693–695 (1986)

Communications in
Mathematical
Physics

Ensemble Average of an Arbitrary Number of Pairs of Different Eigenvalues Using Grassmann Integration

Nazakat Ullah

Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India

Abstract. An identity satisfied by the eigenvalues of a real-symmetric matrix and an integral representation of a determinant using Grassmann variables are used to show that the ensemble average of S different pairs of eigenvalues of a GOE is given by $(-1)^S 2^{-S}\pi^{-1/2}\Gamma(S+\frac{1}{2})$.

1. Introduction

The idea of a matrix ensemble originally introduced by Wigner [1] has been successfully used to study the average properties of the compound nucleus levels [2]. The basic assumption is that each element of a real-symmetric $N\times N$ Hamiltonian matrix has an independent Gaussian distribution. This leads to an eigenvalue distribution known as the Wishart distribution. Because of the appearance of a factor with absolute sign in this distribution, further integrations [3] over the eigenvalues cannot be carried out in a simple way. The purpose of the present work is to show that a simple identity between the eigenvalues and the matrix elements of the Hamiltonian together with a representation of a determinant in terms of Grassmann variables can be used to find ensemble averages of the products of eigenvalues. We shall describe this formulation in Sect. 2. The concluding remarks will be presented in Sect. 3.

2. Formulation

Let us consider a real-symmetric $N\times N$ Hamiltonian matrix, then the joint distribution of its N diagonal and $\frac{1}{2}N(N-1)$ off-diagonal matrix elements can be written as

$$P(\{H_{ij}\})=2^{\frac{1}{4}N(N-1)}\pi^{\frac{1}{4}N(N+1)}\exp(-\mathrm{Tr}\,H^2)\,. \tag{1}$$

The eigenvalues E_i of the Hamiltonian matrix H satisfy the following identity:

$$\prod_{i=1}^{N}(1-\lambda E_i)=\det(1-\lambda H)\,, \tag{2}$$

where λ is a parameter.