Commun. Math. Phys. 104, 611–656 (1986)

Communications in Mathematical Physics

Quaternionic Quantum Field Theory

Stephen L. Adler

The Institute for Advanced Study, Princeton, NJ 08540, USA

Abstract. We show that a quaternionic quantum field theory can be formulated when the numbers of bosonic and fermionic degrees of freedom are equal and the fermions, as well as the bosons, obey a second order wave equation. The theory takes the form of either a functional integral with quaternion-imaginary Lagrangian, or a Schrödinger equation and transformation theory for quaternion-valued wave functions, with a quaternion-imaginary Hamiltonian. The connection between the two formulations is developed in detail, and many related issues, including the breakdown of the correspondence principle and the Hilbert space structure, are discussed.

1. Introduction

A basic theorem [1] in the foundations of quantum mechanics states that a general quantum mechanical system can be represented as a vector space with scalar coefficients drawn from the real, the complex, or the quaternion fields.[1] Standard quantum mechanics and quantum field theory correspond to the complex case, while real quantum mechanics has been analyzed by Stueckelberg [3] and can be shown to reduce back to the complex case. Over the years a number of papers studying the quaternionic case have appeared and some useful mathematical and kinematical results have been obtained [4], but the central problem of finding a viable dynamics for quaternionic quantum theory has remained unsolved. We report progress on this problem in this paper. Specifically, we show[2] that a dynamics for interacting quaternionic quantum fields can be formulated when the numbers of bosonic and fermionic degrees of freedom are equal and the fermions, as well as the bosons, obey a second-order wave equation.

[1] If the requirement of an associative multiplication is dropped, there is a fourth possibility, octonionic quantum mechanics, in which the scalar coefficients form a division algebra [2]. We assume an associative (but not commutative) multiplication in this paper, and so our analysis does not apply to the octonionic case

[2] A brief, partial account of the results of this paper appeared in [5]