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Fock Representations of the Affine Lie Algebra $A_1^{(1)}$

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Abstract. The aim of this note is to show that the affine Lie algebra $A_1^{(1)}$ has a natural family $\pi_{\mu,\nu}$ of Fock representations on the space $\mathbf{C}[x_i, y_j;\ i\in\mathbb{Z}$ and $j\in\mathbb{N}]$, parametrized by $(\mu,\nu)\in\mathbf{C}^2$. By corresponding the highest weight $\Lambda_{\mu,\nu}$ of $\pi_{\mu,\nu}$ to each (μ,ν), the parameter space $\mathbf{C}^2$ forms a double cover of the weight space $\mathbf{C}\Lambda_0\oplus\mathbf{C}\Lambda_1$ with singularities at linear forms of level -2; this number is (-1)-times the dual Coxeter number. Our results contain explicit realizations of irreducible non-integrable highest weight $A_1^{(1)}$-modules for generic (μ,ν).

1. Introduction

It is well known that the basic representations of affine Lie algebras have concrete realizations in terms of vertex operators, and that they have close connections to some areas in mathematical physics such as soliton theory and dual resonance models. But there seems to be only a few results on constructions of highest weight modules other than basic representations even in the case of the affine Lie algebra $A_1^{(1)}$.

In this note our interests are concentrated especially on *non-integrable* representations of $A_1^{(1)}$. We shall show that almost all irreducible non-integrable highest weight $A_1^{(1)}$-modules are realized in a unified way on the Fock space $\mathbf{C}[x_i, y_j;\ i\in\mathbb{Z}$ and $j\in\mathbb{N}]$. Among them, the most interesting case is the case when the representation is of level -2; we shall construct the representation of level -2 on the Fock space $\mathbf{C}[x_i;\ i\in\mathbb{Z}]$, and derive a character formula which is related to the one conjectured by Kac and Kazhdan [5].

2. On the space $\mathbf{C}[x]=\mathbf{C}[x_j; j\in\mathbf{Z}]$ of polynomial functions in x_j's, we introduce the following differential operators:

$$a_j = Y_+(j)x_j - Y_-(j)\frac{\partial}{\partial x_j},$$

$$a_j^* = Y_+(j)\frac{\partial}{\partial x_j} + Y_-(j)x_j,$$