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One Dimensional $1/|j-i|^s$ Percolation Models: The Existence of a Transition for $s \leqq 2$

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Abstract. Consider a one-dimensional independent bond percolation model with p_j denoting the probability of an occupied bond between integer sites i and $i \pm j$, $j \geqq 1$. If p_j is fixed for $j \geqq 2$ and $\lim_{j\to\infty} j^2 p_j > 1$, then (unoriented) percolation occurs for p_1 sufficiently close to 1. This result, analogous to the existence of spontaneous magnetization in long range one-dimensional Ising models, is proved by an inductive series of bounds based on a renormalization group approach using blocks of variable size. Oriented percolation is shown to occur for p_1 close to 1 if $\lim_{j\to\infty} j^s p_j > 0$ for some $s < 2$. Analogous results are valid for one-dimensional site-bond percolation models.

1. Introduction and Main Results

We consider translation-invariant one-dimensional independent site-bond percolation models in which each site $i \in \mathbf{Z}$ is alive (respectively dead) with probability λ (respectively $1-\lambda$) and in which the (non-directed) bond between any distinct $i, j \in \mathbf{Z}$ is occupied (respectively vacant) with probability $p_{|j-i|}$ (respectively $1-p_{|j-i|}$). All the sites and bonds are mutually independent. We will treat both nonoriented and oriented percolation. In ether case the cluster of i, $\mathbf{C}(i)$, consists of those living sites for which there is a path of occupied bonds starting at i, ending at j, and touching only living sites; in particular $i \in \mathbf{C}(i)$ if and only if i is alive. In nonoriented percolation, any such path is allowed; in oriented percolation only paths that move to the right at each step are allowed. Such site-bond models reduce to pure bond models when $\lambda = 1$ and to pure site models when each $p_j = 0$ or 1.

A special case is bond percolation with $\lambda = 1$ and $p_j = 1 - \exp(-\beta |j|^{-s})$ for some s, $\beta \geqq 0$. It is an elementary fact that for $s \leqq 1$, percolation occurs (i.e., $P_\infty \equiv P(\|\mathbf{C}(0)\| = \infty) > 0$, where $\|\mathbf{C}\|$ denotes the number of sites in $\mathbf{C}$) for any $\beta > 0$;

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