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Ground State and Lowest Eigenvalue of the Laplacian for Non-Compact Hyperbolic Surfaces

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Abstract. Let M be a complete Riemannian surface with constant curvature -1, infinite volume, and a finitely generated fundamental group. Denote by $\lambda(M)$ the lowest eigenvalue of the Laplacian on M, and let ψ_M be the associated eigenfunction. We estimate the size of $\lambda(M)$ and the shape of ψ_M by a finite procedure which has an electrical circuit analogue. Using the Margulis lemma, we decompose M into its thick and thin parts. On the compact thick components, we show that ψ_M varies from a constant value by no more than $O(\sqrt{\lambda(M)})$. The estimate for $\lambda(M)$ is calculable in terms of the topology of M and the lengths of short geodesics of M. An analogous theorem of the compact case was treated in [SWY].

Let M be a complete Riemannian surface with constant curvature -1, infinite volume, and a finitely generated fundamental group. The lowest eigenvalue of the Laplacian, $\lambda(M)$, belongs to $(0, 1/4]$ and if $\lambda(M)$ belongs to $(0, 1/4)$, there is a unique positive eigenfunction ψ_M for $\lambda(M)$ of L^2 norm one (see [P1, S, S2]). If one writes $\lambda(M) = D(1-D)$ with $D > 1/2$ then D is the Hausdorff dimension of the limit set of the Fuchsian group representing M. Also the value of ψ_M lifted to the unit disk at a point p is just the (packing) Hausdorff D-measure of the limit set in the metric on rays from p (see [S, S2]).

In this paper we will describe a finite procedure for determining the size of $\lambda(M)$ and the shape of ψ_M. The procedure has an electrical circuit analogue which could, in principle, be used to compute this size and shape.

As a corollary let Γ_ε be the Fuchsian group generated by $z \to -1/z$ and the translation by $2+\varepsilon$. Our Theorem 1 implies the Hausdorff dimension of the limit set of Γ_ε differs from one by the quantity $\sqrt{\varepsilon}$. This estimation implies the derivative of the dimension at the critical value $\varepsilon = 0$ is $+\infty$ (see [P2]).

Furthermore, the estimate has the simple interpretation often observed in dynamically defined Cantor sets on the line, namely that when the dimension is

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