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Unification of Fermion and Boson Stochastic Calculus*

R. L. Hudson[1] and K. R. Parthasarathy[2]

[1] Mathematics Department, University of Nottingham, University Park, Nottingham NG7 2RD, England
[2] Indian Statistical Institute, 7 SJS Sansanwal Marg, New Delhi 110016, India

Abstract. Fermion annihilation and creation processes are explicitly realised in Boson Fock space as functions of the corresponding Boson processes and second quantisations of reflections. Conversely, Boson annihilation and creation processes can be constructed from the Fermion processes. The existence of unitary stochastic evolutions driven by Fermion and gauge noise is thereby reduced to an equivalent Boson problem, which is then solved.

1. Introduction

In [3] we constructed, out of the Boson field commutation relations, a quantum stochastic calculus including an Ito product formula. Existence of solutions of a corresponding class of quantum stochastic differential equations was proved, leading to unitary operator valued processes which, together with their generalisations [4], provide a natural method of dilating quantum dynamical semigroups. In its simplest form the Boson stochastic calculus uses as integrators the Boson annihilation and creation operators corresponding to the indicator function $\chi_{[0,t]}$ in $L^2(\mathbb{R}_+)$, together with the differential second quantisation of the operator of multiplication by $\chi_{[0,t]}$, the latter being called the gauge process.

In [1] a partial analog of this theory was given, in which Fermion second quantisation replaces Boson. The Fermion theory contains that of the Ito-Clifford integral [2] just as the Boson contains that of classical Brownian motion [3]. However no Fermion analog of the gauge process was used, and the unitary processes constructed were artificially restricted to being even with respect to the natural $\mathbb{Z}_2$-grading of Fermion Fock space and an assumed $\mathbb{Z}_2$-grading of the initial space. While this work was being prepared we received a preprint [8] which remedies these deficiencies.

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