Commun. Math. Phys. 104, 445–455 (1986)

Communications in Mathematical Physics

Bounds for the Limiting Variance of the "Heavy Particle" in R^1

Domokos Szász and Bálint Tóth*

Mathematical Institute of the Hungarian Academy of Sciences, V. Reáltanoda u. 13–15, H-1053 Budapest, Hungary

Abstract. We consider a one-dimensional system consisting of a tagged particle of mass M surrounded by a gas of unit-mass hard-point particles in thermal equilibrium. Denoting by Q_t the displacement of the tagged particle, we give lower and upper bounds – independent of M – for $\overline{\lim} E\frac{Q_t^2}{t}$. It results from the proof that the correct nontrivial norming of Q_t – if any – is $\sqrt{t}$.

1. Introduction

Consider the following one-dimensional system of point-like particles: a particle of mass M (the "heavy particle") is surrounded by particles of mass 1 ("light particles") distributed on the line according to a Poisson distribution with density $\varrho = 1$. The velocities of the particles are distributed independently according to Gaussian laws with mean zero: that of the heavy particle with variance $M^{-1/2}$, those of the light ones with variances 1 (Maxwellian distributions with inverse temperature $\beta = 1$). No interaction among the light particles exists and the heavy particle interacts with the light ones through a hard-core potential of radius 0. That is: they collide elastically. It is well known that the dynamics of this system is well defined with probability one and the measure as seen from the heavy particle is stationary.

Let us denote by V_t the velocity and by $Q_t = \int_0^t V_s ds$ the displacement of the heavy particle. It is widely believed that the suitably scaled trajectory of the heavy particle converges to a Brownian motion, but, at present, there is only partial progress in this direction. Before describing it, we make two simple remarks.

Remarks. 1. As far as the behaviour of the heavy particle is concerned only, this system is equivalent with the system of the same particles also assuming elastic collisions between the light particles.

* Work supported by the Central Research Found of the Hungarian Academy of Sciences (Grant No. 476/82)