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A Supersymmetric Transfer Matrix and Differentiability of the Density of States in the One-Dimensional Anderson Model

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Abstract. Let $H = -\Delta + V$ on $l^2(\mathbb{Z})$, where $V(x)$, $x \in \mathbb{Z}$, are i.i.d.r.v.'s with common probability distribution ν. Let $h(t) = \int e^{-itv} d\nu(v)$ and let $k(E)$ be the integrated density of states. It is proven: (i) If h is n-times differentiable with $h^{(j)}(t) = O((1+|t|)^{-\alpha})$ for some $\alpha > 0$, $j = 0, 1, \ldots, n$, then $k(E)$ is a C^n function. In particular, if ν has compact support and $h(t) = O((1+|t|)^{-\alpha})$ with $\alpha > 0$, then $k(E)$ is C^∞. This allows ν to be singular continuous. (ii) If $h(t) = O(e^{-\alpha|t|})$ for some $\alpha > 0$ then $k(E)$ is analytic in a strip about the real axis.

The proof uses the supersymmetric replica trick to rewrite the averaged Green's function as a two-point function of a one-dimensional supersymmetric field theory which is studied by the transfer matrix method.

1. Introduction

The one-dimensional Anderson model is given by the random Hamiltonian $H = H_0 + V$ on $l^2(\mathbb{Z})$, where

$$(H_0 u)(x) = \tfrac{1}{2}(u(x+1) + u(x-1))$$

and $V(x)$, $x \in \mathbb{Z}$, are independent identically distributed random variables with common probability distribution ν. We will denote by h its characteristic function, i.e., $h(t) = \int e^{-itv} d\nu(v)$.

Let Λ be an interval in $\mathbb{Z}$, we will denote by H_Λ the operator H restricted to $l^2(\Lambda)$ with boundary condition $u(x) = 0$ for x not in Λ.

The integrated density of states, $k(E)$, is defined by

$$k(E) = \lim_{|\Lambda| \to \infty} \# \{\text{eigenvalues of } H_\Lambda \leqq E\}.$$

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