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The Distributional Borel Summability and the Large Coupling Φ^4 Lattice Fields

E. Caliceti, V. Grecchi, and M. Maioli

Dipartimento di Matematica, Università di Modena, I-41100 Modena, Italy

Abstract. Following 't Hooft we extend the Borel sum and the Watson-Nevanlinna criterion by allowing distributional transforms. This enables us to prove that the characteristic function of the measure of any $g^{-2}\Phi^4$ finite lattice field is the sum of a power series expansion obtained by fixing exponentially small terms in the coefficients. The same result is obtained for the trace of the double well semigroup approximated by the n^{th} order Trotter formula.

1. Introduction

Borel summability has by now become an important tool largely applied in many fields of mathematical physics (e.g. see [7, 17]). On the other hand it has also become clear that certain problems do not fulfill all the requirements for Borel summability. The simplest and best known counterexample is probably the double well quantum mechanical model with Hamiltonian: $p^2+x^2(1-gx)^2$ [3, 8]. Other "non-Borel" series are expected in quantum field theory because of the presence of "renormalons" [12]. The lack of Borel summability of a real divergent series is evident when the coefficients have asymptotically constant sign [3, 17].

A few years ago 't Hooft showed [10] that a simplified double well model is Borel summable only in some generalized sense, allowing distributions in the transform. Following 't Hooft we define here a class of distributional Borel sums extending the ordinary Borel-Le Roy ones, and correspondingly we extend the Watson-Nevanlinna criterion. This way we prove that the characteristic function of the measure of a $g^{-2}\Phi^4$ finite volume field is the sum of its power series expansion obtained by fixing exponentially small terms in the coefficients. More precisely the characteristic functions have the following asymptotic expansions:

$$C(z,g)\sim\sum_k a_k(z,g^{-2})g^k,\quad \text{where}\quad a_k(z,\gamma)=\sum_{m=0}^{N} a_m^k(z)e^{-m\gamma},$$

and $C(z,\gamma,g)\sim\sum\limits_k a_k(z,\gamma)g^k$ is the Borel sum of the asymptotic series for small g and any positive γ, in particular for $\gamma=g^{-2}$.