Commun. Math. Phys. 104, 87–102 (1986)

Communications in
Mathematical
Physics

Multidimensional Random Walks in Random Environments with Subclassical Limiting Behavior

Richard Durrett*

Department of Mathematics, University of California, Los Angeles, California**, USA

Abstract. In this paper we will describe and analyze a class of multidimensional random walks in random environments which contain the one dimensional nearest neighbor situation as a special case and have the pleasant feature that quite a lot can be said about them. Our results make rigorous a heuristic argument of Marinari et al. (1983), and show that in any $d < \infty$ we can have (a) X_n is recurrent and (b) $X_n \sim (\log n)^2$.

1. Introduction

In this paper we will describe some new results concerning multidimensional random walks in random environments (hereafter abbreviated RWRE). To put our results in perspective we will start by describing the one dimensional nearest-neighbor model and stating the results that we will generalize. Let $p_x \in (0, 1)$, $x \in Z$ be a stationary sequence [i.e., the distribution of $(p_{x-j}, \dots p_{x+j})$ is independent of x]. This sequence is the environment. We think of it as being generated at time 0 and then fixed for all time while a particle wanders around on Z moving as a discrete time Markov Chain with transition probabilities $p(x, x+1) = p_x$, $p(x, x-1) = 1 - p_x$, i.e., if it is at x it flips a coin with these probabilities to determine its next position. If we let X_n be the position of the particle at time n then X_n is our RWRE. When the environment is nonrandom (and by stationarity constant) X_n is a random walk, so with results about these systems in mind, the first question we would like to ask is "Will X_n return to 0 infinitely often?". If $p_x \equiv p$ then the answer is yes if and only if $p = 1/2$, so one might guess than the answer in the random case is $Ep_x = 1/2$. This is wrong, however, but the correct answer is not difficult to determine and is given by

* AMS "Mid Career" Fellowship 1984–1986. Research also partially supported by NSF Grant MCS 83-00836

** Address after July 1, 1985: Department of Math., Cornell University, Ithaca, NY 14853, USA