Commun. Math. Phys. 104, 21–36 (1986)

Communications in Mathematical Physics

Bounds on the Trajectories of a System of Weakly Coupled Rotators

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Abstract. In this paper we study a classical mechanical system of weakly coupled rotators on a one-dimensional lattice. Such systems are of interest in statistical mechanics. We prove that for any site in the system there is a "large" set of initial conditions for which there exists a canonical change of variables such that the trajectory of that site, in the transformed system, is essentially indistinguishable from that of an integrable system for a long (but finite) time. Alternatively, the trajectory of this site lies very close to a torus in the phase space of the system for times very long in comparison with the typical period of the unperturbed rotators. All the estimates in this theory are *independent* of the number of degrees of freedom in the system. We propose this mechanism as an explanation of certain numerical experiments.

1. Introduction and Main Results

In this paper we analyze the behavior of trajectories in the classical mechanical system with Hamiltonian

$$H(I, \phi) = \tfrac{1}{2} I \cdot I + \varepsilon \sum_{i=1}^{N} \cos(\phi_i - \phi_{i+1}), \tag{1.1}$$

where $I \in \mathbb{R}^N$ and $I \cdot I$ is the usual Euclidean inner product. More specifically given $j \in \{1, \ldots, N\}$ we will be interested in the behavior of $\{I_j(t), \phi_j(t)\}$, for long but finite times. In [8] Galgani and LoVecchio have studied a system of weakly coupled oscillators numerically. Looking at the trajectory of a single oscillator in the chain they find that as they increase the energy/mode of the system a transition occurs from a state in which the trajectory of the oscillator is confined to a relatively small region about the trajectory it would follow if uncoupled from the rest of the chain, to a state where the trajectory wanders over a large part of the phase plane. Furthermore they find that the energy/mode at which this transition occurs is the same whether the number of degrees of freedom, $N = 20$ or 100. Further numerical

† Supported in part by NSF Grant DMS-8403664