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Central Limit Theorem for Additive Functionals of Reversible Markov Processes and Applications to Simple Exclusions

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Abstract. We prove a functional central limit theorem for additive functionals of stationary reversible ergodic Markov chains under virtually no assumptions other than the necessary ones. We use these results to study the asymptotic behavior of a tagged particle in an infinite particle system performing simple excluded random walk.

Introduction

In a recent work Lebowitz and Spohn [4] proved that diffusion of color for "mechanically" identical particles with two colors as well as convergence to local equilibrium in the hydrodynamical limit were related to the asymptotics, after proper rescaling, of the movements of one or more tagged particles of the system.

Since for the moment purely mechanical systems seem to be out of reach, several models which are to lesser or greater extent stochastic have been proposed. In [3] Kipnis et al. considered the case of a one dimensional system of hard rods with stochastic collisions. In this paper we consider the case of the so-called simple exclusion process.

The intuitive description of the process is the following: Infinitely many particles move on $\mathbb{Z}^d$ according to a simple random walk with exponential (mean one) holding time at each site and jump law $p(x)$ which is symmetric, i.e. satisfies $p(x) = p(-x)$ for all x's. However if a particle attempts a transition to a site already occupied, the jump is suppressed.

The key remark is that, due to invariance of the mechanics under translations, the evolution of the rest of the medium *seen from an observer sitting on a tagged particle* follows a Markovian evolution (which possesses many reversible probabilities) and that the movement of the tagged particle (the observer) in the absolute frame is in a certain sense "driven" by this Markov process.

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