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Gauge Covariant Theory of the Generating Operator. I

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Abstract. A gauge covariant formulation of the generating operator (Λ-operator) theory for the Zakharov-Shabat system is proposed. The operator $\tilde{\Lambda}$, corresponding to the gauge equivalent system in the pole gauge is explicitly calculated. Thus the unified approach to the nonlinear Schrödinger-type equations based on Λ is automatically reformulated with the help of $\tilde{\Lambda}$ for the Heisenberg ferromagnet-type equations. Consequently, it is established that the conserved densities for the Heisenberg-ferromagnet-type equations are polynomial in $S(x)$ and its x-derivatives. Special attention is paid to the interrelation between the hierarchies of symplectic structures corresponding to the above mentioned families of gauge-equivalent equations. It is shown that the geometrical properties of the conjugated operator Λ^* are gauge-independent.

1. Introduction

It is well known that the inverse scattering method (ISM) relates to a given linear problem $L(q, \lambda)$, where $q(x)$ denotes a set of coefficient functions and λ the spectral parameter, a class of exactly solvable nonlinear evolution equations (NLEEs). A paradigm of such a linear problem is the so-called Zakharov-Shabat system [1, 2]:

$$L(q, \lambda)\psi = \left(i\frac{d}{dx} - q - \lambda\sigma_3\right)\psi = 0,$$

$$q = q_+\sigma_+ + q_-\sigma_-, \qquad q_\pm(x) \in \mathbb{C}, \tag{1.1}$$

$$\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

A number of physically important NLEEs, such as the nonlinear Schrödinger equation (NLSE):

$$i\sigma_3 q_t + q_{xx} + 2q\langle q, q\rangle = 0, \qquad \langle q, q\rangle = \tfrac{1}{2}\operatorname{tr} q^2, \tag{1.2}$$

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