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First-Order Transitions in Spherical Models: Finite-Size Scaling

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Abstract. Finite-size behavior near the first-order phase boundary of ferromagnetic spherical models is investigated for block- and cylinder-shaped systems in d dimensions. The bulk thermodynamic singularities are rounded and, asymptotically for large size, obey appropriate scaling laws. Both short-range interactions and long-range couplings, decaying like $1/r^{d+\sigma}$ with $\sigma > 0$, are analyzed: the short-range results agree precisely with a recently developed scaling theory for $O(n)$ symmetric systems in the limit $n \to \infty$. More generally, the scaling functions are universal, depending only on σ. Explicit aspects of the shape and interactions enter only in the "spin wave" or "Goldstone mode" contributions which appear, technically, as "corrections to scaling." An appendix analyzes the truncation error in the approximation, by many-fold sums, of multivariate integrals with integrands diverging like $[\sum_j a_j \theta_j^2]^{-\lambda}$ as $\boldsymbol{\theta} \to 0$.

1. Introduction

The spherical model of a ferromagnet [1] was devised by Kac and is of interest, in particular, because it can be solved in closed form in the thermodynamic limit [2–5]. For dimensionality, d, exceeding $d=2$, it exhibits a phase transition if the interactions are of short range. However, it is soluble also for long-range, power-law interactions. Furthermore, it exhibits a nontrivial variation of the critical exponents with dimensionality and interaction decay, which makes it a valuable tool in studying bulk and surface critical behavior [6, 7]. The model is also tractable when one or more of the linear dimensions are large but finite. Thus it has been employed to test the theory of finite-size scaling [8, 9] which describes the rounding or distortion of the bulk critical singularities that must occur in a system with restricted dimensions. These developments have been reviewed by Barber [10]; see also [11–13] for some more recent studies.

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