

Localization Formulas, Superconnections, and the Index Theorem for Families

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Abstract. In this paper, we give a new proof of the localization formulas of Berline and Vergne [9] and Duistermaat and Heckman [18]. When interpreted in the framework of Atiyah [2], the probabilistic heat equation proof of the Index Theorem given in our paper [12] appears as the rigorous infinite dimensional version of this new proof of the localization formulas in finite dimensions. The results of Quillen [25] on superconnections are briefly presented. The heat equation proofs [15] of the Index Theorem for families are described. It is shown that in this framework, the superconnections formalism is the operator theoretic description of integration along the fiber in the loop space.

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