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Equilibrium Fluctuations for Interacting Brownian Particles

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Abstract. We consider an infinitely extended system of Brownian particles interacting by a pair force $-\text{grad}\, V$. Their initial distribution is stationary and given by the Gibbs measure associated with the potential V with fugacity z. We assume that V is symmetric, finite range, three times continuously differentiable, superstable, and positive and that the fugacity is small in the sense that $0 \leqq z \leqq 0.28/e \int dq(1-e^{-V(q)})$. In addition a certain essential self-adjointness property is assumed. We prove then that the time-dependent fluctuations in the density on a spatial scale of order ε^{-1} and on a time scale of order ε^{-2} converge as $\varepsilon \to 0$ to a Gaussian field with covariance $\chi \int dq g(q)(e^{(\rho/2\chi)\Delta|t|} f)(q)$ with ρ the density and χ the compressibility.

1. Introduction

A system of interacting Brownian particles is governed by the equations of motion

$$\frac{d}{dt} x_j(t) = -\tfrac{1}{2} \sum_{i \neq j} \text{grad}\, V(x_j(t) - x_i(t)) + \frac{d}{dt} \omega_j(t), \tag{1.1}$$

$j = 1, 2, \ldots$. The particles interact by a pair force, $-\text{grad}\, V$, and are driven by white noises $(d/dt)\omega_j(t)$ independently for each particle. Physically such a system is best realized by an aqueous suspension of polystyrene spheres with roughly 500 Å diameter [1, 2]. The spheres are charged, on the order of several hundred elementary charges, and interact therefore by the screened Coulomb potential V. In addition the spheres interact hydrodynamically through pressure forces mediated by the water. This interaction is neglected in (1.1) which seems to be a good approximation for volume fractions less than 0.01. Under this condition the random force on a polystyrene sphere due to the bombardment by water molecules has a correlation time of the order of 10^{-10} s. During this time span a sphere typically moves only 0.3 Å. Therefore to suppress in (1.1) the velocities of the particles is a very reasonable approximation.

Such a system of Brownian particles in equilibrium is investigated experimentally by means of light scattering which measures directly the structure function $S(k,t)$. Here k is related to the scattering angle and t is the delay time. Theoretically $S(k,t)$ is given by the spatial Fourier transform of the density–density correlation