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The U(1) Higgs Model

I. The Continuum Limit

C. King[†][*]

Department of Physics, Harvard University, Cambridge, MA 02138, USA

Abstract. By using rigorous renormalization group methods we construct the continuum limit of the finite-volume lattice U(1) Higgs model in two and three dimensions. The method relies on a proof of the convergence of the effective action.

1. Introduction

Recently, renormalization group methods have been used to study lattice regularizations of Euclidean quantum field theories. In particular, Balaban has proved ultra-violet stability for the finite volume lattice Higgs model in three dimensions [Ba 1–4], obtaining bounds independent of the lattice spacing. In this paper we construct the continuum limit of this model in two and three dimensions. In a succeeding paper, the infinite volume limit will also be constructed, and some of the Osterwalder-Schrader axioms verified. This model was constructed previously in two dimensions by Brydges et al. [BFS 1–3].

The U(1) Higgs model is an interacting theory of a vector field $A_\mu(x)$ coupled in a gauge covariant way to a N-component scalar field $\phi(x)$. The classical (Euclidean) action of the model in d dimensions is

$$S(A,\phi)=\int d^dx\left\{1/4\sum_{\mu,\nu=1}^{d}|F_{\mu\nu}(x)|^2+1/2\sum_{\mu=1}^{d}|D_\mu\phi(x)|^2+1/2m^2|\phi(x)|^2+\lambda|\phi(x)|^4\right\}. \tag{1.1}$$

The field strength tensor is $F_{\mu\nu}(x)=\partial_\mu A_\nu(x)-\partial_\nu A_\mu(x)$, and the covariant derivative of the scalar field is

$$(D_\mu\phi)_i(x)=\partial_\mu\phi_i(x)-eA_\mu(x)\,(q\phi)_i(x)\,. \tag{1.2}$$

The coupling constants of the theory are e, λ and q is an antisymmetric $N\times N$ matrix. The action (1.1) is invariant under local gauge transformations: define

$$A_\mu^\chi=A_\mu(x)-\partial_\mu\chi(x)\,,\qquad \phi^\chi(x)=\exp[-eq\chi(x)]\phi(x)\,, \tag{1.3}$$

† Present address: Department of Physics, Princeton University, Princeton, NJ 08544, USA

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