Commun. Math. Phys. 102, 605–647 (1986)

Communications in Mathematical Physics

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Homogeneous Kähler Manifolds: Paving the Way Towards New Supersymmetric Sigma Models

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Abstract. Homogeneous Kähler manifolds give rise to a broad class of supersymmetric sigma models containing, as a rather special subclass, the more familiar supersymmetric sigma models based on Hermitian symmetric spaces. In this article, all homogeneous Kähler manifolds with semisimple symmetry group G are constructed, and are classified in terms of Dynkin diagrams. Explicit expressions for the complex structure and the Kähler structure are given in terms of the Lie algebra $\mathfrak{g}$ of G. It is shown that for compact G, one can always find an Einstein-Kähler structure, which is unique up to a constant multiple and for which the Kähler potential takes a simple form.

1. Introduction and Summary of Results

Non-linear sigma models are natural candidates for effective low-energy theories, and they play an important rôle in our present understanding of symmetry breaking. In fact, whenever a field-theoretical model exhibits a (global) symmetry under a Lie group G which is spontaneously or dynamically broken down to a closed subgroup K, then independently of the details of the underlying dynamics, the associated Goldstone bosons are, in the low-energy sector, described by the non-linear sigma model on the homogeneous space G/K[1]. A similar scenario applies when all models are replaced by their supersymmetric extensions – at least as long as supersymmetry remains unbroken.

Now it is well known that the definition of a supersymmetric non-linear sigma model (with ordinary $N=1$ supersymmetry in four dimensions or with extended $N=2$ supersymmetry in two dimensions) requires the corresponding "field space" to be a Kähler manifold [1]. In fact, in four dimensions and in terms of superfields, the Lagrangian of the model (to be integrated over superspace) is simply the so-

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1 The term "homogeneous space" is synonymous for "coset space," and similarly, the term "Hermitian symmetric space" is synonymous for "symmetric Kähler manifold"