Commun. Math. Phys. 102, 585–604 (1986)

Communications in
Mathematical
Physics

Integration on Supermanifolds and a Generalized Cartan Calculus

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Abstract. A suggestion by Berezin for a method of integration on supermanifolds is given a precise differential geometric meaning by assuming that a supermanifold is the total space of a fibre bundle with connection. The relevant objects for integration are identified as suitable horizontal/vertical projections of hyperforms. The latter are generalizations of differential forms having both covariant and contravariant indices. The exterior calculus of these projected hyperforms is developed, analogously to the Cartan calculus, by introducing appropriate derivations and determining their commutators, respectively anticommutators.

1. Introduction

The concepts of rigid and curved superspace have turned out to be of great importance in current research on supersymmetry and supergravity. As originally introduced by Salam and Strathdee [1] superspace has, besides the coordinates x^μ ($\mu=0,\ldots,3$), which are commuting (even, bosonic), additional anticommuting (odd, fermionic) coordinates $\theta^\alpha(\alpha=1,\ldots,4)$. Superfields are functions depending on these variables and encode both bosonic and fermionic fields by means of a Taylor expansion in the odd variables. Integration of superfields with respect to the odd variables is given an operational definition by the Berezin integration rules [2]. Also a (super)-tensor calculus and the notions of (super)-connection, -torsion and -curvature are used frequently in the physics literature [3, 4].

Many authors have investigated how to make these more or less heuristic ideas mathematically rigorous. Rogers [5] introduced the concept of a D_0+D_1 dimensional supermanifold modelled over $B_L^{(D_0,D_1)}$, a space obtained from a Grassmann algebra B_L. Several modifications of her approach have been proposed [4, 6–8]. The construction of tensor bundles on supermanifolds broadly resembles the procedure for C^∞ manifolds.

What is still lacking is a fully satisfactory theory of integration on supermanifolds mimicking the Berezin integration rules. For C^∞ manifolds the relevant