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Integrability of Two Interacting *N*-Dimensional Rigid Bodies

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Abstract. A new class of integrable Euler equations on the Lie algebra so$(2n)$ describing two n-dimensional interacting rigid bodies is found. A Lax representation of equations of motion which depends on a spectral parameter is given and complete integrability is proved. The double hamiltonian structure and the Lax representation of the general flow is discussed.

1. Introduction

The Euler equations on the SO(n) Lie group, which describe the rotation of a free n-dimensional rigid body about a fixed point, have the following set of n quadratic, mutually commuting, integrals of motion

$$K_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\ell_{ij}^2}{\alpha_i - \alpha_j} \quad (i = 1, \ldots, n), \tag{1.1}$$

where ℓ_{ij} are the angular momentum dynamical variables and α_j, $j = 1, \ldots, n$ are real parameters. Integrals of the form (1.1) have been for the first time considered by Uhlenbeck (see [1]) for the motion of a mass point on a unit sphere under the influence of a harmonic potential. But they play a special role in the motion of an n-dimensional rigid body, since the Manakov [2] integrable system corresponds to the hamiltonian

$$H = \frac{1}{2} \sum_{i=1}^{n}{}' \beta_i K_i = \frac{1}{2} \sum_{i<j} \frac{\beta_i - \beta_j}{\alpha_i - \alpha_j} \ell_{ij}^2, \tag{1.2}$$

where β_j are real parameters and the summation is taken over all pairs $i<j$.

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