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Perturbation Theory and Non-Renormalizable Scalar Fields

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Abstract. We study how to set up systematic summation rules that could permit us to interpret the divergent expressions arising in the perturbation theory of $:P(\varphi):_d$ when one does not allow any renormalization besides the usual coupling constants, mass and wave function renormalizations.

1. Introduction

Our main result is that it is possible to express the Schwinger functions (or the effective potentials) as formal power series of objects which we call "form factors" which, although divergent to all orders of perturbation theory if the cut-off N is removed, obey to all orders a formal equation which retains its meaning as $N \to \infty$.

We show that if the formal equation admits a solution verifying suitable bounds, then the formal power series for the Schwinger functions in terms of the form factors is bounded to all orders.

Hence there is the possibility of giving a meaning to perturbation theory of non-renromalizable interactions without introducing infinitely many new counterterms, but rather introducing infinitely many new constants, the form factors, which however are not independent but are related by an equation (which may or may not have some non-trivial solution).

We restrict ourselves to the case of renormalizable (but not superrenormalizable) or non-renormalizable polynomial interactions in integer dimension $d \geqq 3$. The superrenormalizable cases would require a separate treatment. It is conceivable that something like the results of this paper hold for some non-polynomial interactions (like sine-Gordon field in two dimensions): however the whole problem should be studied starting again from scratch.

While the ideas involved in this paper are partly already in the literature (see Parisi, 1973, 1975; Symanzik, 1973, and references therein) the bounds that we

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