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Quantum R Matrix for the Generalized Toda System

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Abstract. We report the explicit form of the quantum R matrix in the fundamental representation for the generalized Toda system associated with non-exceptional affine Lie algebras.

1. Introduction

It has been known for some time that the Yang–Baxter (YB) equations play a crucial rôle in classical and quantum integrable systems (see e.g. [1]). The structure of the classical YB equation is now fairly well understood [2–3]. In ref. [3] a classification of non-degenerate solutions related to simple Lie algebras is given, subject to the unitarity condition. Unfortunately such classification is yet unavailable in the quantum case. One of the consequences of [3] is that the trigonometric solutions, up to certain equivalence, are finite in number, and that they allow a neat description in terms of Dynkin diagrams. An immediate question would be whether it is possible to quantize all these solutions. The most typical ones among them are the classical solutions associated with the generalized Toda system (GTS). In this paper we report on the corresponding quantum solutions for the case of non-exceptional affine Lie algebras.

To be more specific, we consider the solutions $r(x)$ of the classical YB equation

$$[r^{12}(x), r^{13}(xy)] + [r^{12}(x), r^{23}(y)] + [r^{13}(xy), r^{23}(y)] = 0 \tag{1.1}$$

for the GTS of type $A_n^{(1)}$, $B_n^{(1)}$, $C_n^{(1)}$, $D_n^{(1)}$, $A_{2n}^{(2)}$, $A_{2n-1}^{(2)}$ and $D_{n+1}^{(2)}$, as given in Eq. (2.3), (3.1–4). Here the notations are standard: $r(x)$ is a $\mathfrak{G}\otimes\mathfrak{G}$-valued rational function, $\mathfrak{G}$ being a finite dimensional simple Lie algebra, and $r^{12}(x) = r(x)\otimes I$, etc. The problem is to find an $R(x) = R(x, \hbar)$ containing an arbitrary parameter $\hbar$, such that (i) it satisfies the quantum YB equation

$$R^{12}(x)R^{13}(xy)R^{23}(y) = R^{23}(y)R^{13}(xy)R^{12}(x), \tag{1.2}$$

and (ii) as $\hbar \to 0$,

$$R(x, \hbar) = \kappa(x, \hbar)(I + \hbar r(x) + \cdots) \tag{1.3}$$

holds with some scalar $\kappa(x, \hbar)$. In contrast to the classical case (1.1), the quantum