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On a C^*-Algebra Approach to Phase Transition in the Two-Dimensional Ising Model. II

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Abstract. We investigate the states ϕ_β on the C^*-algebra of Pauli spins on a one-dimensional lattice (infinitely extended in both directions) which give rise to the thermodynamic limit of the Gibbs ensemble in the two-dimensional Ising model (with nearest neighbour interaction) at inverse temperature β. We show that if β_c is the known inverse critical temperature, then there exists a family $\{v_\beta : \beta \neq \beta_c\}$ of automorphisms of the Pauli algebra such that

$$\phi_\beta = \begin{cases} \phi_0 \circ v_\beta, & 0 \leqq \beta < \beta_c \\ \phi_\infty \circ v_\beta, & \beta > \beta_c. \end{cases}$$

1. Introduction

We consider the Ising Hamiltonian on a two-dimensional lattice, infinitely extended in all directions, with nearest neighbour interactions and zero field. Thus the problem is classically set in the commutative C^*-algebra $C(\mathscr{P}) = \bigotimes_{\mathbb{Z}^2} \mathbb{C}^2$ of all continuous functions on the configuration space $\mathscr{P} = \{\pm 1\}^{\mathbb{Z}^2}$. The transfer matrix method allows us to transform the model to a non-commutative algebra $\mathscr{A}^P = \bigotimes_{\mathbb{Z}} M_2$ in one dimension less. More precisely, for each inverse temperature β, suppose $\langle \cdot \rangle_\beta$ is the equilibrium state for the classical system obtained as the thermodynamic limit of the Gibbs ensembles on the configuration space $\mathscr{P}$ using free boundary conditions. Then there is for each β, a map $F \to F_\beta$ from the local observables in $C(P)$ into the Pauli or quantum algebra $\mathscr{A}^P$ such that $\langle F \rangle_\beta = \phi_\beta(F_\beta)$. Thus any classical correlation or expectation value can be computed using a knowledge of the Pauli algebra alone. The main result of [3] was the following:

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