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BRS Cohomology and Topological Anomalies

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Abstract. The occurrence of non-abelian anomalies in gauge theories and gravitation, first discovered via perturbative techniques, is now completely explained from the mathematical point of view by means of the family index theorem of Atiyah and Singer. Here we make contact between this approach and BRS cohomology, by showing that they yield the same non-abelian anomalies, provided a certain restriction to "local" functionals is not introduced from the very beginning. In particular, this solves the "unicity" problem for this kind of anomalies. Local BRS cohomology is still relevant for the abelian case.

1. Introduction

As it is well known, the conservation of certain fermionic currents in gauge theories does not survive quantization, giving rise to the so-called anomalies. These were first discovered via Feynman diagram techniques [1], but have been successively studied by a variety of tools.

First, a cohomological interpretation was found [2] thanks to the nilpotency of the BRS operator δ. A functional $a=a(A,\omega)$ (locally depending on the gauge potential A and linear in the ghost ω) is called an anomaly if it satisfies the Wess-Zumino consistency condition $\delta a=0$, but there is no local functional $\Lambda_{\text{loc}}(A)$ such that $-\delta\Lambda_{\text{loc}}(A)=a$. In physical terms this implies that there is no redefinition $\Gamma\to\Gamma+\Lambda_{\text{loc}}$ of the "effective action" Γ which cancels the anomaly itself. Clearly, the problem is of cohomological nature, and a proper mathematical set up involves the cohomology of the Lie algebra of infinitesimal gauge transformations with local functionals of the gauge potentials as coefficients [3]. We call this cohomology the "local" BRS-cohomology.

Soon after the cohomological nature of anomalies was understood, their relations with the secondary characteristic classes of Chern and Simons was

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