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Localization in General One Dimensional Random Systems, I. Jacobi Matrices

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Abstract. We consider random discrete Schrödinger operators in a strip with a potential $V_\omega(n, \alpha)$ (n a label in $\mathbb{Z}$ and α a finite label "across" the strip) and V_ω an ergodic process. We prove that $H_0 + V_\omega$ has only point spectrum with probability one under two assumptions: (1) The *conditional* distribution of $\{V_\omega(n, \alpha)\}_{n=0,1,\text{all}\,\alpha}$ conditioned on $\{V_\omega\}_{n\neq 0,1;\text{all}\,\alpha}$ has an absolutely continuous component with positive probability. (2) For a.e. E, no Lyaponov exponent is zero.

1. Introduction

This is the second of three papers exploiting ideas of Kotani [11] to understand localization of random Schrödinger operators. In the basic paper of the series with Wolff [20], we combined ideas of Aronszajn [1]–Donoghue [7] and an abstract analog of averaged boundary condition results of Carmona [2]–Kotani [11] to prove localization in the Anderson model (potential given by i.i.d.'s). In this paper, we discuss more general discrete random Schrödinger operators, and in a companion paper with Kotani [12], we discuss the continuum case. Our main result is stated in the abstract, but we would emphasize also our new and, we feel, especially transparent way of going from positive Lyaponov exponents to the critical condition $\int (x-E)^{-2} d\mu_0(x) < \infty$ on spectral measures (see Sects. 3 and 4).

After completing the research we describe here, we learned that Delyon, Levy and Souillard [4–6], also motivated by Kotani [11], had proven results very close to those we find here. They follow Kotani's approach more clodely than we do; in particular, generalized eigenfunction expansions play a crucial role in their arguments, while they do not herein.

We also learned that Fröhlich et al. [21] use their study of localization in ν-dimensions (at large couplings or energy) to study the one-dimensional case at arbitrary coupling and energy.

The fundamental result of Wolff–Simon [20] is the following:

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