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Finiteness of Ricci Flat Supersymmetric Non-linear σ-Models

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Abstract. Combining the constraints of Kähler differential geometry with the universality of the normal coordinate expansion in the background field method, we study the ultraviolet behavior of 2-dimensional supersymmetric non-linear σ-models with target space an arbitrary riemannian manifold M. We show that the constraint of $N=2$ supersymmetry requires that all counterterms to the metric beyond one-loop order be cohomologically trivial. It follows that such supersymmetric non-linear σ-models defined on locally symmetric spaces are super-renormalizable and that $N=4$ models are on-shell ultraviolet finite to all orders of perturbation theory.

1. Introduction

Non-linear σ-models are the quantum field theories of maps from spacetime into a riemannian manifold M. A few years ago, it was shown [1] that the ultraviolet properties of supersymmetric non-linear σ-models can be studied using the geometrical properties of Kähler manifolds. In particular, it was shown [1, 2] that supersymmetric σ-models on 2-dimensional spacetimes with target space M a Ricci flat manifold are ultraviolet finite up to three-loop order, and it was conjectured that such models were ultraviolet finite to all orders of perturbation theory. Specific examples of Ricci flat manifolds were shown in [3] to result in supersymmetric σ-models finite to all orders of perturbation theory, but these examples were not sufficiently general to imply the result for arbitrary Ricci flat manifolds. In this paper we will provide a proof that $N=4$ supersymmetric σ-models are indeed finite to all orders of perturbation theory. We regard such theories as consequently likely to admit exact solutions. We also discuss extensions of our methods here to the case of arbitrary Ricci flat manifolds. Our intent is to illustrate as cleanly as possible the qualitative features underlying the finiteness of these models. On this basis we occasionally resort to compelling arguments rather than to truly rigorous proofs to establish certain intermediate results.

Part of the motivation for clarifying these issues is provided by recent progress [4] in string theory which suggests that certain 10-dimensional supersymmetric