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Some Results on the Quantum Dynamics of a Particle in a Markovian Potential

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Abstract. We consider the quantum dynamics of a particle in a time dependent potential $V(t)$, assuming it to be a Markovian random function of time. We derive a formula for the density matrix at time t averaged over the realisations of the potential. We then obtain a kind of RAGE theorem for the time evolution of compact observables, and some information on the phase space behaviour of the system.

I. Introduction and Results

Since the pioneering work of Anderson [1] on localisation of electronic states in a static disordered lattice, random time independent Schrödinger operators have become increasingly popular in solid state physics to describe effects of impurities and thermal disorder in crystals. By now such models are also fairly well understood mathematically, see for example [2] and [3]. On the other hand time dependent random Hamiltonians received much less attention, probably because of the intrinsic difficulties encountered when dealing with non-autonomous systems. We mention some works (see for example [4]) on the $\delta(t)$-correlated gaussian random potential, typically a problem of the form:

$$i\partial_t\psi_t = (-\Delta + V_t)\psi_t \text{ on the lattice } \mathbb{Z}^\nu, \tag{1.1}$$

where Δ is the discrete Laplacian (or some tight binding Hamiltonian), and $V_t(x)$ a gaussian random field of mean zero and covariance

$$\langle V_t(x)V_s(y)\rangle = g\delta(t-s)\delta_{x,y}.$$

Such models can be explicitly solved to give a finite diffusion constant, i.e.

$$0 < \lim_{t\to\infty}\left\langle \frac{1}{t}\sum_x x^2|\psi_t(x)|^2 \right\rangle = D < \infty,$$

and thus a finite electrical conductivity. Another interesting special case is to take the potential V_t in (1.1) as a Markovian random function of time. Such potentials can be constructed in the following way: choose a homogeneous Markov process $\xi(t)$ on