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Large-Time Behavior of the Broadwell Model of a Discrete Velocity Gas

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Abstract. We study the behavior of solutions of the one-dimensional Broadwell model of a discrete velocity gas. The particles have velocity ± 1 or 0; the total mass is assumed finite. We show that at large time the interaction is negligible and the solution tends to a free state in which all the mass travels outward at speed 1. The limiting behavior is stable with respect to the initial state. No smallness assumptions are made.

1. Introduction

Broadwell [1] studied the structure of shocks in a model of rarefied gas in which the particles travel with speed c in either direction along a coordinate axis. If particles traveling in opposite directions collide, they are equally likely to move in each of the three coordinate directions after collision, with velocities of opposite sign. Other collisions can only lead to an exchange of velocities. If $N_1^+(x, y, z, t)$ is the density of particles with velocity $(c, 0, 0)$, N_1^- the density with velocity $(-c, 0, 0)$, and similarly for $N_2^\pm$, $N_3^\pm$, the resulting equations are

$$D_t N_1^+ + c D_x N_1^+ = \frac{\sigma}{3}(N_2^+ N_2^- + N_3^+ N_3^- - 2N_1^+ N_1^-),$$

$$D_t N_1^- - c D_x N_1^- = \frac{\sigma}{3}(N_2^+ N_2^- + N_3^+ N_3^- - 2N_1^+ N_1^-),$$

etc., where σ is the frequency of collision. (Such discrete velocity models of a gas were introduced by Maxwell; see [5] for a survey.)

Here we consider the special case of one-dimensional motions in which the N's are independent of y, z, and furthermore $N_2^+ = N_2^- = N_3^+ = N_3^-$. Setting $N_1^+ = v(x, t)$, $N_1^- = w(x, t)$, $N_2^+ = z(x, t)$ and rescaling the variables so that $c = 1$, $\sigma = 3/2$, we can

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