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The Incompressible Limit in Nonlinear Elasticity

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Abstract. The incompressible limit in nonlinear elasticity is shown to fall under the theory of singular limits of quasilinear symmetric hyperbolic systems developed by Klainerman and Majda. Specifically, initial-value problems for a family of hyperelastic materials with stored energy functions

$$W\left(\frac{\partial x}{\partial X}\right) = W_\infty\left(\frac{\partial x}{\partial X}\right) + \lambda^2 w\left(\det\frac{\partial x}{\partial X}\right)$$

are considered, where X and x are reference and deformed coordinates respectively. Under the assumption that the elasticity tensor

$$A^{ij}_{kl} \equiv \frac{\partial^2 W_\infty}{\partial\left(\frac{\partial x^i}{\partial X^k}\right)\partial\left(\frac{\partial x^j}{\partial X^l}\right)}$$

is positive definite near the identity matrix and that $w''(1) > 0$, the following results are proven for appropriate initial data:

i) existence of solutions of the corresponding evolution equations on a time interval independent of λ as $\lambda \to \infty$, and ii) convergence as $\lambda \to \infty$ of the solutions to a solution of the incompressible elastodynamics equations.

1. Introduction

The incompressible limit in fluid dynamics has received considerable attention in the last few years [2, 6, 7, 11, 15]. The basic result, which has been proven in various contexts, is that slightly compressible fluid flow is close to incompressible flow, even though the equations for the latter are related to those for the former via a singular limit. This justifies the use of the incompressible flow equations for certain real fluids that are actually slightly compressible.

A similar situation occurs in elasticity: certain types of rubber strongly resist changes in volume, and so are often modelled as incompressible solids. This procedure has been justified for elastostatics by Rostamian [10]; our goal here is to justify it, at least in certain contexts, for hyperelastic dynamics by applying the

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