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Monodromy Fields on $\mathbb{Z}^2$

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Abstract. Lattice monodromy fields are defined and the massive scaling regime is controlled.

Introduction

In this paper we introduce a family of lattice fields in two dimensions which are related to and include the two dimensional Ising field. The introduction of these fields was inspired by work of Sato, Miwa, and Jimbo (SMJ) on the Riemann–Hilbert problem [17, II] and the analysis of the Euclidean Dirac equation in [17, III] and [17, IV]. In a sense explained in Sect. 2, the fields introduced here are lattice analogues of the continuum fields used by SMJ in [17, IV].

I believe these lattice fields are interesting for a number of reasons. First, by working on the lattice and controlling the scaling limit one may make mathematically precise sense of the Euclidean wave functions used to such good effect in SMJ [17, IV]. Much of the present paper is devoted to laying the foundation for an analysis of the scaled correlations carried out along the lines of the analysis of the Ising correlations in [9]. Second, the numerous analogues of continuum structures on the lattice suggests the possibility of a discrete "SMJ" analysis of the lattice correlations. Work of McCoy, Wu and Perk [3–5, 11], which demonstrates that the Ising model correlations may be expressed in terms of the solutions to non-linear partial difference equations leaves one with little doubt for the future success of such a program for the more general models considered here. The second section of this paper is devoted to a cursory look at the lattice wave functions, the finite difference equation they satisfy and the characterization of a finite dimensional family of solutions with prescribed branch points and monodromy (Theorem 2.0). The difference between the lattice case and the continuum case is instructive and I believe the role of the τ-function is considerably clarified in the lattice formulation (see the proof of Theorem 2.0). A third reason I am interested in these fields concerns critical scaling limits. By such a scaling limit we mean the large scale asymptotics for the correlations at the critical temperature (zero mass limit). It is something of a scandal that despite the enormous amount of work that has been done on the two dimensional Ising correlations there is still no definitive account of the large scale