Commun. Math. Phys. 102, 139–155 (1985)

Communications in
Mathematical
Physics

Construction of a Non-Trivial Planar Field Theory with Ultraviolet Stable Fixed Point

Giovanni Felder

Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland

Abstract. We study a ϕ_4^4 planar euclidean quantum field theory with propagator $1/p^{2-\varepsilon/2}$, $\varepsilon>0$. With the help of the tree expansion of Gallavotti and Nicolò [1], this non-renormalizable theory is shown to have a non-trivial ultraviolet-stable fixed point at negative coupling constant. The vicinity of the fixed point is discussed.

1. Introduction

We consider a ϕ_4^4 theory with propagator $1/p^{2-\varepsilon/2}$, $\varepsilon>0$. The coupling constant has a mass dimension ε making the theory non-renormalizable and very similar to a $\phi_{4+\varepsilon}^4$ model. In particular, one expects to find an ultraviolet stable fixed point at negative coupling. For the planar approximation (i.e. if we keep only the planar Feynman graphs in perturbation theory) we prove that this fixed point does exist, yielding a scale invariant field theory. An expansion about this fixed point provides then a two-parameter family of continuum theories.

The method we used is the tree expansion of Gallavotti and Nicolò [1] which is based on the ideas of Wilson [2] (see also [3], where a similar way of implementing Wilson's ideas in perturbation theory is presented). The tree expansion is an expansion in powers of the running coupling constants on all scales, which is finite order by order (and convergent in planar theories) even if the theory is non-renormalizable [4]. The running coupling constants are related to one another by recursion relations (the flow equations) and the problem is reduced to the simpler one of finding a solution to the flow equations which is in the convergence domain of the tree expansion. If ε is small a two-parameter family of solutions is shown to exist in the planar diagram approximation. This gives a rigorous meaning to the ε-expansion [2] for the $N\to\infty$ limit of an $N\times N$ matrix model with $\mathrm{tr}\,\phi^4$ interaction.

The paper which is largely self-contained is organized as follows: Section 2 is a summary of the methods in [1] applied to the present model: the tree expansion of the effective potentials and the flow equations are formulas (2.14), (2.19). The