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The Lipatov Argument for ϕ_3^4 Perturbation Theory

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Abstract. We extend to φ_3^4 the work of S. Breen on the leading behavior at large order of φ_2^4 perturbation theory. Using a phase space expansion to obtain new estimates on the high energy behavior of φ_3^4 Feynman graphs, and a rigorous semiclassical expansion, we prove that the radius of convergence of the Borel transform of the pertubative series for φ_3^4 Euclidean field theory is the one computed by the Lipatov method.

I. Introduction

The Lipatov method is a formal steepest descent method for finding the asymptotic behavior at large order of perturbation series in the Euclidean path integral formulation of quantum field theory. Following early work by Bender and Wu [1] and Lam [2], the first calculations by Lipatov [3] were restricted to massless φ^{2N} field theory in dimension $\frac{2N}{N-1}$. The method was extensively developed by Brézin, Le Guillou and Zinn-Justin [4] to compute the large order behavior of general bosonic theories. After arguments by Parisi and 't Hooft [5] it was realized that the result should hold only for superrenormalizable theories. Yet even there a general rigorous justification of the Lipatov method has not been given. Let us summarize the work done in this direction and the difficulties.

For simplicity we limit ourselves in this paper to the perturbative expansion for the pressure of the massive one-component φ^4 model in dimensions 1, 2, or 3, in which it is superrenormalizable. We rescale also the bare mass to be 1. Extensions to arbitrary mass, to N-component vector models and to general Schwinger functions are easy, once this simple case has been rigorously understood, and we do not discuss them here.

The partition function of the model in a volume Λ is defined by constructive field theory [6, 7] as:

$$Z_X(g) = \int e^{-gV(\varphi) + \text{counterterms}} d\mu_X(\Lambda), \tag{1.1}$$

in which $V(\varphi) = \int_\Lambda \varphi^4(x) d^dx$, $\Lambda = [-T/2, T/2]^d$ and $X = p$ (periodic) or D (Dirichlet) specifies the two possible types of boundary conditions that we will