Commun. Math. Phys. 102, 47–58 (1985)

Communications in Mathematical Physics

A Uniqueness Theorem for Systems of Interacting Polymers

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Abstract. In this paper we study analyticity properties of the pressure of general polymer systems by application of Ruelle's theorem on the zeros of Asano contracted polynomials to the partition function. Then we prove a uniqueness theorem for the equilibrium state of the investigated class of classical particle systems for a variety of finite range interactions.

1. Introduction

Lattice models are still an effective tool to solve phase transition problems in classical statistical mechanics. In the centre of interest we find models without restrictions. In these models the particles of the system can occupy one point of the lattice. But there were moreover a lot of attempts to develop models for extended particles that cover more than one point without penetrating each other. Especially hard rod models have been widely used to explain the existence of liquid crystals. Thus the theory of monomer-dimer systems is extensively elaborated. We refer the reader to [6] for a brief historical sketch that shows, how monomer-dimer systems are related to physics and chemistry.

In [5, 6] the absence of a phase transition is proved for translationally invariant systems of *noninteracting* monomers and dimers. In [1, 7] Abraham and Heilmann respectively Heilmann and Lieb proved with the help of reflection positivity arguments the existence of a phase transition for some systems of *interacting* monomers and dimers. We will mention here also the papers [8, 9, 12], that contain results for monomer-dimer systems, but their assumptions on the interactions are very artificial.

Gruber and Kunz incorporated more complicated with respect to their geometry particles into their model in [5]. They showed the absence of a phase transition for *noninteracting polymers* and thus generalized the results of [6]. We are going to introduce and to investigate a very general model for *interacting polymers* on the lattice $\mathbb{Z}^d$. Under the assumption of $\mathbb{Z}^d$-invariance for such polymer systems a variational principle for the pressure $P(J)$ of an interaction J is