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Gross-Neveu Model Through Convergent Perturbation Expansions

K. Gawędzki[1,2]

C.N.R.S., I.H.E.S., F-91440 Bures-sur-Yvette, France

A. Kupiainen[2]

Research Institute for Theoretical Physics, University of Helsinki, Helsinki, Finland

Abstract. We construct a continuum limit for the effective low energy Lagrangians of the Gross-Neveu model in two euclidean dimensions by showing that they are related to each other through convergent perturbation expansions. This provides a rigorous control of the ultraviolet problem in a renormalizable quantum field theory.

1. Introduction

It has been observed, back in the fifties, that in quantum field theories with fermions, the perturbation expansion is better behaved than in purely bosonic ones. Indeed, it was noted [1] that for QED with finite ultraviolet (UV) and infrared (IR) cutoffs the expansion in fact converges, in sharp contrast with bosonic theories where the perturbation series is only asymptotic. However, the situation is much less clear in renormalizable theories of fermions once the UV cutoff has been removed and the renormalization subtractions performed. The nice property of the cutoff theory seems to be lost as is indicated by the conjectured presence of renormalon singularities in the Borel transform of the renormalized perturbation series of any asymptotically free model, regardless of statistics [2]. In the present paper, we show that important traces of the convergence for cutoff fermions remain when the cutoff is removed. In fact, we construct a local renormalizable asymptotically free theory of fermions by a successive application of convergent perturbation expansions, one per each limited range of momentum fluctuations.

We consider the Gross-Neveu models [3, 4] in two (euclidean) dimensions: the perturbatively renormalizable, asymptotically free field theories of Dirac fermions $\psi^i_\alpha(x)$, $\alpha=0,1$, $i=1,\dots,N$, with $N>1$ internal [U(N) "flavor"] symmetry indices, and the action given by

$$S=\int dx[\bar{\psi}i\partial\!\!\!/\psi-g(\bar{\psi}\psi)^2]\,, \tag{1}$$

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