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Local Existence of the Borel Transform in Euclidean Massless Φ_4^4

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Abstract. We extend the methods of [1] to prove large order estimates on the renormalized Feynman amplitudes of massless Φ_4^4 euclidean field theory, at non-exceptional momenta. The Borel transform of the perturbative series is analytic in a disk centered at the origin of the complex plane. This result is a step towards the rigorous investigation of the infra-red singularities in the Borel plane, for theories containing massless particles, like the gauge theories.

I. Introduction

In [1], the large order behaviour of the perturbation series was rigorously investigated, for the renormalized massive Φ_4^4 field theory. The n^{th} order term for any Schwinger function at fixed external momenta was bounded by $K^n n!$, where K is a constant (depending on the external momenta). This implies the existence and analyticity of the Borel transform of the perturbative series, at least in a disk of the complex plane centered at the origin. This result has been rederived recently by other methods [2].

To extend the work of [1] to field theories containing massless particles is a non-trivial and interesting problem for several reasons. The renormalization scheme with subtractions at zero external momenta becomes ill-defined due to infra-red singularities in the amplitudes. Therefore, one should renormalize with subtractions at some intermediate energy scale μ, and the renormalized amplitudes are finite only at non-exceptional momenta. The structure of the corresponding renormalization operator, ensuring that the physical mass of the particles effectively vanishes, gets more complicated and the resulting structure is richer. In particular, varying the energy scale of the subtraction point, one recovers renormalization group equations, and interesting phenomena like the infra-red asymptotic freedom of the Φ_4^4 theory can be analyzed. More generally, the study of theories with massless particles, especially the analytic structure of their Borel transform, is certainly relevant to the program of rigorous construction of gauge theories, both in their complete version or in approximations like the $N \to \infty$ planar theories.