Commun. Math. Phys. 101, 501–538 (1985)

Communications in Mathematical Physics

First Order Phase Transitions in Lattice and Continuous Systems: Extension of Pirogov-Sinai Theory*

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Abstract. We generalize the notion of "ground states" in the Pirogov-Sinai theory of first order phase transitions at low temperatures, applicable to lattice systems with a finite number of periodic ground states to that of "restricted ensembles" with equal free energies. A restricted ensemble is a Gibbs ensemble, i.e. equilibrium probability measure, on a restricted set of configurations in the phase space of the system. When a restricted ensemble contains only one configuration it coincides with a ground state. In the more general case the entropy is also important.

An example of a system we can treat by our methods is the q-state Potts model where we prove that for q sufficiently large there exists a temperature at which the system coexists in $q+1$ phases; q-ordered phases are small modifications of the q perfectly ordered ground states and one disordered phase which is a modification of the restricted ensemble consisting of all "perfectly disordered" (neighboring sites must have different spins) configurations. The free energy thus consists entirely of energy in the first q-restricted ensembles and of entropy in the last one.

Our main motivation for this work is to develop a rigorous theory for phase transitions in continuum fluids in which there is no symmetry between the phases, e.g. the liquid-vapour phase transition. The present work goes a certain way in that direction.

1. Introduction

In 1936, Peierls [1] invented an argument to show that the Ising model on a d-dimensional lattice, $d \geqq 2$, with nearest neighbor ferromagnetic interactions has spontaneous magnetization at low enough temperatures. The system can exist in either a + or a − phase: the signature of a first order phase transition. Dobrushin [2] and Griffiths [3] later made the argument mathematically precise. This

* Supported in part by NSF Grant N^r DMR81-14726-02