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Formation of Singularities in Three-Dimensional Compressible Fluids

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Abstract. Presented are several results on the formation of singularities in solutions to the three-dimensional Euler equations for a polytropic, ideal fluid under various assumptions on the initial data. In particular, it is shown that a localized fluid which is initially compressed and outgoing, on average, will develop singularities regardless of the size of the initial disturbance.

This article presents a number of results on formation of singularities in solutions to the three-dimensional compressible Euler equations for a polytropic, ideal fluid. The results, described precisely in the following section, show that C^1 solutions to the equations do not exist for all time, under various restrictions on the initial data.

Theorems one and two, below, deal with the case of "large" data which essentially means that the initial flow velocity must be supersonic in some region relative to the sound speed at infinity. (The initial data is constant outside a bounded set.) Singularity formation is detected as the disturbance overtakes the wave front (presumably as a shock wave) forcing the front to propagate with supersonic speed. The method, which is a refinement of [10], applies equally well in one and two space dimensions.

Our main result, Theorem 3, shows that a fluid will develop singularities if, on average, it is slightly compressed and out-going near the wave front. The proof borrows some important technical points from an earlier result on nonlinear wave equations in three space dimensions [9].

Although the extensive one-dimensional theory [2, 5, 7, 8] strongly suggests such results, the customary method of characteristics has not proved tractable in higher dimensions. (John [3] has recently used characteristics to prove blow-up for a three-dimensional scalar wave equation with spherical symmetry.) Our approach involves the use of averaged quantities, thereby avoiding for the most part, local analysis of the solution. For classical solutions, these averages satisfy certain differential inequalities, solutions of which have finite life span.

We do not address the problem of singularity formation in three-dimensional incompressible fluids. It is possible to approximate a compressible flow with an

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