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On Symmetric Solutions of the Relativistic Vlasov–Poisson System*

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Abstract. Spherically symmetric solutions to the Cauchy problem for the relativistic Vlasov–Poisson system are studied in three space dimensions. If the energy is positive definite (the plasma physics case), global classical solutions exist. In the case of indefinite energy, "small" radial solutions exist in the large, but "large" data solutions (those with negative energy) will blow-up in finite time.

I. Introduction

The motion of a mono-charged collisionless plasma is described by the Vlasov–Maxwell (VM) system of equations:

$$(\mathrm{VM})\quad \begin{cases} f_t + v\cdot\nabla_x f + \gamma(E + v\times B)\cdot\nabla_v f = 0\\ E_t = \nabla\times B - j\\ B_t = -\nabla\times E\\ \nabla\cdot E = \rho,\quad \nabla\cdot B = 0.\end{cases}$$

Here E and B are the Maxwell electric and magnetic fields and $f = f(x, v, t)$ ($x\in\mathbb{R}^3$, $v\in\mathbb{R}^3$, $t \geqq 0$) is a scalar function describing the density in phase space. The charge and current densities are given by

$$\rho = \rho(x,t) = \int_{\mathbb{R}^3} f(x,v,t)\,dv,$$

$$j = j(x,t) = \int_{\mathbb{R}^3} v f(x,v,t)\,dv,$$

and $\gamma = \pm 1$. The case $\gamma = +1$ is the plasma physics case, and $\gamma = -1$ is the stellar dynamics case.

The Cauchy problem is to solve (VM) for all $t > 0$ with given initial values for E,

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