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The Stochastic Geometry of Invasion Percolation

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Abstract. Invasion percolation, a recently introduced stochastic growth model, is analyzed and compared to the critical behavior of standard d-dimensional Bernoulli percolation. Various functions which measure the distribution of values accepted into the dynamically growing invaded region are studied. The empirical distribution of values accepted is shown to be asymptotically unity above the half-space threshold and linear below the point at which the expected cluster size diverges for the associated Bernoulli problem. An acceptance profile is defined and shown to have corresponding behavior. Quantities related to the geometry of the invaded region are studied, including the surface to volume ratio and the volume fraction. The former is shown to have upper and lower bounds in terms of the above defined critical points, and the latter is bounded above by the probability of connection to infinity at the half-space threshold. Provided that the critical regimes of Bernoulli percolation possess their anticipated properties, as is known to be the case in two dimensions, these results verify numerical predictions on the acceptance profile, establish the existence of a sharp surface to volume ratio and show that the invaded region has zero volume fraction. Large-time asymptotics are analyzed in terms of the probability that the invaded region accepts a value greater than x at time n. This quantity is shown to be bounded below by $h(x)\exp[-c(x)n^{(d-1)/d}]$ for x above threshold, and to have an upper bound of the same form for x larger than a particular value (which coincides with the threshold in $d=2$). For two dimensions, it is also established that the infinite-time invaded region is essentially independent of initial conditions.

1. Introduction

Invasion percolation is a stochastic growth model that was introduced and studied numerically by two independent groups [1, 2]. (See also [28] for some related

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